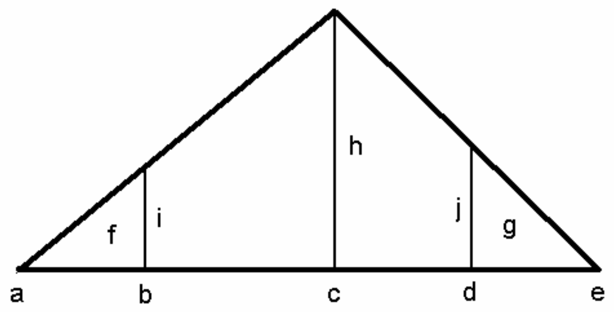
TRIGEN Documentation



Given: Points b, c and d; Areas f and g. Area of the triangle is 1.

To be computed: Points a and e.

Triangle area is 1: [i] $h(e-a)=2 \Leftrightarrow h=\dfrac{2}{e-a}$

[ii] $\dfrac{h}{c-a}=\dfrac{i}{b-a} \Leftrightarrow i=h\dfrac{b-a}{c-a}=\dfrac{2(b-a)}{(e-a)(c-a)}$

[iii] $\dfrac{h}{e-c}=\dfrac{j}{e-d} \Leftrightarrow j=h\dfrac{e-d}{e-c}=\dfrac{2(e-d)}{(e-a)(e-c)}$

[iv] $2g = j(e-d)=\dfrac{2(e-d)^2}{(e-a)(e-c)} \Rightarrow a=e-\dfrac{(e-d)^2}{g(e-c)}$

[v] $2f = i(b-a)=\dfrac{2(b-a)^2}{(e-a)(c-a)}$

Substituting a in [v] and putting everything on one side gives:

$$\left\{b-e+\frac{(e-d)^2}{g(e-c)}\right\}^2 - f\left\{e-e+\frac{(e-d)^2}{g(e-c)}\right\}\left\{c-e+\frac{(e-d)^2}{g(e-c)}\right\}=0$$

$\Leftrightarrow$

$$\frac{\left((b-e)g(e-c)+(e-d)^2\right)^2}{\left(g(e-c)\right)^2} - f\,\frac{(e-d)^2}{g(e-c)}\,\frac{(c-e)(g(e-c)+(e-d)^2}{g(e-c)}=0$$

$\Leftrightarrow$ [g≠0 and e≠c]

$$\left(g(b-c)(e-c)+(e-d)^2\right)^2 - f(e-d)^2\left((e-d)^2 - g(e-c)^2\right) = 0$$

$\Leftrightarrow$

$$g^2(b-e)^2(e-c)^2 + 2g(b-e)(e-c)(e-d)^2 + (e-d)^4 - f(e-d)^4 - fg(e-d)^2(e-c)^2 = 0$$

$\Leftrightarrow$

$$\begin{aligned}
&e^4(fg - f + 1 - 2g + g^2) \\
&+ e^3(-2fgd - 2fgc - 4d + 4fd + 2gc + 4gd + 2gb - 2g^2c - 2g^2b) \\
&+ e^2(fgd^2 + 4f\,\mathrm{gcd} + fgc^2 - 6fd^2 + 6d^2 - 4\,\mathrm{gcd} - 2gd^2 - 2gbc - 4gbd + g^2c^2 + 4g^2bc + g^2b^2) \\
&+ e(-2f\,\mathrm{gcd}^2 - 2fgc^2d + 4d^3f - 4d^3 + 2\,\mathrm{gcd}^2 + 4gbcd + 2gbd^2 - 2g^2bc^2 - 2g^2b^2c) \\
&+ (fgc^2d^2 - fd^4 + d^4 - 2gbcd^2 + g^2b^2c^2 = 0
\end{aligned}$$

The correct solution for e can now be calculated with a Newton iteration with a starting value > e, for example with the start value

$$d + \frac{d-c}{(1-g)^2}$$

If g=0 and f>0 then switch f and g, b and d and later the solution a and e.